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1996 J. Phys.: Condens. Matter 8 313

(http://iopscience.iop.org/0953-8984/8/3/010)

J. Phys.: Condens. Matter **8** (1996) 313–327. Printed in the UK

Electrical properties of a two-dimensional electron gas under a general one-dimensional periodic magnetic field

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Received 1 May 1995, in final form 18 July 1995

Abstract. We have given a detailed investigation of the energy spectrum and the electrical properties of a two-dimensional electron gas modulated by a general form of one-dimensional periodic magnetic field along the x direction, by generalizing the theory of Peeters and Vasilopoulos and that of Xue and Xiao on the magnetic modulation in the lowest order of approximation of Fourier transformation. The presence of the magnetic modulation lifts the degeneracy of the Landau levels, which are broadened into bands, and leads to Weiss-like oscillations in the magnetoresistance. The oscillations in ρ_{xx} (and the modulation correction $\Delta\rho_{xx}$) and ρ_{yy} (and $\Delta\rho_{yy}$) are out of phase, while $\Delta\rho_{xy}$ oscillates in phase with $\Delta\rho_{xx}$. The amplitude of oscillation of the modulation correction $\Delta\rho_{xx}$ is much larger than those of $\Delta\rho_{yy}$ and $\Delta\rho_{xy}$. We also find the surprising result that, while the Hall resistance displays quantized plateaux, the transport across the magnetic barriers can be nearly dissipationless. The contribution of high-frequency components of Fourier transformation is obvious at high fields and is negligible at low fields.

1. Introduction

Since the discovery of the oscillations in magnetoresistance of the high-mobility two-dimensional electron gas (2DEG) modulated by a one-dimensional (1D) weak periodic potential, discovered due to Weiss *et al* [1], the study of these novel oscillations (also called Weiss oscillations) by different theoretical models and experimental methods [2–14] has attracted much interest from physicists and experimentalists. In a perpendicular magnetic field and a weak 1D potential modulation, Weiss oscillations in the magnetoresistance tensor $\rho_{\mu\nu}$ are periodic in $1/B$ in the same way as the Shubnikov–de Haas (SdH) oscillations, but with a larger period depending on both the modulation period a and the square root $\sqrt{n_e}$ of the areal electron density of 2DEG, in contrast with the linear dependence (on n_e) of SdH oscillations. The amplitude of these novel oscillations has a weak dependence on the temperature in contrast with the sensitive temperature dependence in SdH oscillations. Physically, Weiss oscillations in $\rho_{\mu\nu}$ can be interpreted as a consequence of the commensurability between the length scales of the modulation period a and the cyclotron electron radius $R_N = \sqrt{2N+1}l$ at the Fermi energy, where $l = \sqrt{\hbar/eB}$ and $N = n_F$ is the Landau level index [2–14], whereas SdH oscillations come from the resonance between l and the Fermi wavelength λ_F of the 2DEG [9].

A similar system of great interest is the 2DEG modulated by magnetic field, which has also been discussed by several workers [15–17]. By depositing a series of stripes

0953-8984/96/030313+15$19.50

of magnetic material on top of a high-mobility $Al_xGa_{1-x}As$–GaAs heterojunction, using modern lithography techniques, we get a 2DEG modulated by the 1D periodic magnetic field. In the lowest-order approximation of Fourier transformation, $\mathbf{B} = [B_0 + B_1 \cos(Kx)]\hat{\mathbf{e}}_x$, the results of [15–17] state that magnetoresistance also has two kinds of oscillation: the novel oscillation related to the modulation and the SdH oscillations similar to those of a 2DEG under potential modulation. The aim of this paper is to give a detailed calculation of the magnetoconductivity tensor components for a more general form of the magnetic modulation field, including the higher order of Fourier components of modulation, to the second order of modulation strength.

We organize the paper as follows. In sections 2 and 3, we calculate the one-particle energy spectrum and the density of states (DOS) in the regime of the first-order quantum perturbation theory, which is the starting point of the following sections. The electrical conductivity tensors are calculated in section 4. In section 5, we give the numerical results on the conductivity and resistivity tensors and a detailed discussion of the results. Finally, our conclusion and some remarks are given in section 6.

2. Energy spectrum

We consider a 2DEG lying in the (x, y) plane with a lateral weak periodic modulated magnetic field (the modulation being taken along the x direction) $\mathbf{B} = (B_0 + B_1(x))\hat{\mathbf{e}}_x$, where the oscillating part $|B_1| \ll B_0$. Using the Landau gauge for the vector potential, we take

$$\mathbf{A} = \mathbf{A}_0 + \mathbf{A}_1 = (0, A_0(x) + A_1(x), 0) \tag{1}$$

with $A_0(x) = B_0 x$ and

$$A_1(x) = \sum_g A_1(g) \exp(\mathrm{i}gx) \tag{2}$$

where $g = pK = p2\pi/a$, $p = \pm 1, \pm 2, \ldots$, and a is the spatial modulation period. With the assumption that $A_1(x)$ is a real function, we have $A_1^*(g) = A_1(-g)$, and equation (2) may be rewritten as follows:

$$A_1(x) = \sum_{p=1}^{\infty} 2\,\mathrm{Re}[A_1(g_p) \exp(\mathrm{i}g_p x)]. \tag{3}$$

Without losing generality, to match the above representation with the lowest Fourier term approximation $A_1(x) = (B_1/K)\sin(Kx)$, we may take the following form of $A_1(g)$:

$$A_1(g_p) = \frac{B_1}{2\mathrm{i} f(g_p)} \tag{4}$$

with $f(g_p)$ having the dimension of $g_1 (= K)$, and $f(K) = K$.

The one-particle Hamiltonian of the system is

$$\begin{aligned} H &= \frac{1}{2m^*}[p + e\mathbf{A}]^2 + V_I \\ &= \frac{1}{2m^*}[p + e\mathbf{A}_0]^2 + \frac{1}{m^*}(p_y + eB_0 x)eA_1(x) + \frac{1}{2m^*}(eA_1(x))^2 + V_I \\ &= H_0 + H^1 + V_I \end{aligned} \tag{5}$$

where m^* and p are the effective mass and momentum operator, respectively, of the electron and

$$H_0 = \frac{1}{2m^*}[p + e\mathbf{A}_0]^2 \tag{6}$$

$$H^1 = \frac{1}{m^*}(p_y + eB_0x)eA_1(x) + \frac{1}{2m^*}(eA_1(x))^2 \tag{7}$$

and $V_I(\mathbf{r}) = \sum_j u(\mathbf{r} - \mathbf{R}_i)$ is the effective two-dimensional (2D) scattering potential of the randomly distributed impurities (located at $\mathbf{R}_j$) in the plane of the 2DEG.

In the absence of modulation and impurities, the normalized eigenfunctions of H_0 in equation (6) are given by

$$\psi^0_{nk_y} = \frac{1}{\sqrt{L_y}}\exp(\mathrm{i}yk_y)\Phi_n(x + x_0) \tag{8}$$

corresponding to the eigenvalues $E^0_n = (n + \frac{1}{2})\hbar\omega_c$ (with the cyclotron frequency $\omega_c = eB_0/m^*$), degenerate with respect to the wavevector k_y (in the y direction), where $\Phi_n(x+x_0)$ are the normalized wavefunctions of the 1D harmonic oscillator centred at $x_0 = l^2k_y$, with the minimum cyclotron radius $l = (\hbar/eB_0)^{1/2}$, and L_y the width of the 2DEG in the y direction.

In the perturbation theory of quantum mechanics, the first-order wavefunctions and the corresponding eigenvalues are

$$|nk_y\rangle^1 = |nk_y\rangle^0 + \sum_{m\neq n}\frac{H^1_{n,m}}{E^0_n - E^0_m}|mk_y\rangle^0 \tag{9}$$

and

$$E_{nk_y} = (n + \tfrac{1}{2})\hbar\omega_c + \sum_{p=1}^{\infty}\epsilon_{n,p}\cos(pKx_0). \tag{10}$$

In order to obtain the explicit expressions of $H^1_{n,m}$ and $\epsilon_{n,p}$, we use the formula given in [10] for an arbitrary 2D Fourier component:

$$\begin{aligned}{}^0\langle n'k'_y|\exp(\mathrm{i}\mathbf{q}\cdot\mathbf{r})|nk_y\rangle^0 &= \delta_{k'_y,k_y+q_y}\exp\left[-\frac{\mathrm{i}}{2}l^2q_x(k'_y + k_y)\right]\left(\frac{m!}{M!}\right)^{1/2}\mathrm{i}^{|n'-n|}\\ &\times\left(\frac{(q_x + \mathrm{i}q_y)}{q}\right)^{n-n'}e^{-Q/2}Q^{|n'-n|/2}L^{(|n'-n|)}(Q)\end{aligned} \tag{11}$$

where $Q = \frac{1}{2}l^2q^2$, and $q = \sqrt{q_x^2 + q_y^2}$; $L_n(Q)$ is the associated Laguerre polynomial; m and M are the minimum and maximum respectively, of n and n'. Neglecting the higher-order perturbation term in H^1, we have the matrix elements

$$\begin{aligned}H^1_{n,n'} &= {}^0\langle nk_y|H^1|n'k_y\rangle^0\\ &= \tfrac{1}{2}\hbar\omega_1\sum_{p=1}^{\infty}\frac{pK}{f(pK)}\left\{[1 - |n - n'|u^{-1}p^{-2}]L_m^{(|n-n'|)}(p^2u) - \frac{2}{p^2}\frac{\partial}{\partial u}L_m^{(|n-n'|)}(p^2u)\right\}\\ &\quad\times\left(\frac{m!}{M!}\right)^{1/2}\exp\left(-\frac{p^2u}{2}\right)(p^2u)^{|n-n'|/2}\{\mathrm{Re}(\mathrm{i}^{|n-n'|}\exp(\mathrm{i}pKx_0))\}\end{aligned} \tag{12}$$

and

$$\epsilon_{n,p} = \tfrac{1}{2}\hbar\omega_1\frac{pK}{f(pK)}\exp\left(-\frac{p^2u}{2}\right)[L^1_n(p^2u) + L^1_{n-1}(p^2u)] \tag{13}$$

where $\omega_1 = eB_1/m^*$ and $u = \frac{1}{2}K^2l^2$. $H^1_{n,m}$ and $\epsilon_{n,p}$ are the first-order perturbation quantities and the $p = 1$ terms in equations (12) and (13) coincide with the former results of Peeters and Vasilopoulos [17] in the lowest order of Fourier transformation. From equation (10) we know that the eigenvalues depend on the wavevector k_y and the magnetic modulation has lifted the degeneracy of Landau levels with different wavevectors k_y. The Landau levels broaden into bands, whose width oscillate with the field B_0, the modulation period a and the Landau level index n. Owing to the symmetry of the problem, the wavevector k_y remains a good quantum number, as in the unmodulated case.

In the absence of modulation, ψ^0_{n,k_y} are localized states in the (x, y) plane, i.e. ${}^0\langle nk_y|v_x|nk_y\rangle^0 = 0$ and ${}^0\langle nk_y|v_y|nk_y\rangle^0 = 0$ for all eigenstates while, in the modulated system, the k_y degeneracy of Landau levels has been lifted. To the order of $\hbar\omega_1$, we have

$$ {}^1\langle nk_y|v_x|nk_y\rangle^1 = 0 \tag{14} $$

$$ {}^1\langle nk_y|v_y|nk_y\rangle^1 = -\frac{2u}{\hbar K}\sum_{p=1}^{\infty} p\epsilon_{n,p}\sin\left[\frac{2pu}{K}k_y\right]. \tag{15} $$

The $p = 1$ term of equation (15) corresponds to equation (6) of [17].

The first-order wavefunctions $|nk_y\rangle^1$, different from $|n, k_y\rangle^0$, are localized in the x direction while extended in the y direction, which would lead to important consequences for electrical transport properties, as discussed in section 4.

3. Density of states

A 2DEG system has a different energy spectrum with magnetic modulation from that without; this may also be reflected in the DOSs. In the unmodulated case, $B_1(x) = 0$, the DOS of the system is $D(E) = \sum_{nk_y}\delta(E - E^0_n) = (1/2\pi l^2)\sum_n \delta(E - E^0_n)$, which is expressed for the unit surface of the 2DEG. In this case, the DOS is a series of sharp lines, corresponding to different energy levels $E^0_n = (n + \frac{1}{2})\hbar\omega_c$. For each Landau level, there are $1/2\pi l^2$ states, independent of the index n. In the modulated case, the DOSs take the following form:

$$ \begin{aligned} D(E) &= \sum_{nk_y}\delta(E - E_{n,k_y}) \\ &= \frac{1}{2\pi a}\sum_{n=0}^{\infty}\int_0^{a/l^2} \mathrm{d}k_y\,\delta(E - E_{n,k_y}). \end{aligned} \tag{16} $$

The DOS is broadened into bands with different band widths corresponding to different Landau levels. The number of available states in each Landau level is still $1/2\pi l^2$. The averaging magnitude of the DOS of a specific Landau level will be inversely proportional to the band width. So narrow bands lead to a large peak value of the DOS. As a result, the quantities proportional to the magnitude of the DOS are 180° out of phase with the quantities proportional to the band width of the Landau level.

In summary, the DOS has mainly two kinds of oscillating property. The first comes from the oscillations of the band width, as discussed in section 2. This kind of oscillation exists only in the modulated 2DEG and corresponds to the Weiss oscillations. The second is the usual oscillation of the DOS at the Fermi energy. As the magnetic field increases, the lower Landau levels sweep through the Fermi energy E_F, subsequently resulting in the oscillation of the DOS at E_F. This corresponds to the SdH oscillation. Therefore the oscillating properties of the DOS will have profound effects on the transport quantities of the 2DEG in the case of modulation.

In the practical system, there always exists randomly distributed impurities as introduced in equation (5). For simplicity, let us assume that $V_I(r)$ varies rapidly within the length scale of the cyclotron radius l, i.e.

$$V_I(\mathbf{r}) = V_0 \sum_i \delta(\mathbf{r} - \mathbf{R}_i) \tag{17}$$

where V_0 is the strength of the scattering impurities. Starting from the elastic transition rate formula

$$W_{nk_y,n'k'_y} = \frac{2\pi}{\hbar} |\langle nk_y|V_I|n'k'_y\rangle|^2 \delta(E_{n,k_y} - E_{n',k'_y}) \tag{18}$$

we get the impurity broadening of the Landau levels, $\Gamma_n = \Gamma = (V_0^2 n_i/\pi l^2)^{1/2}$, which is independent of the Landau level number. n_i is the areal density of impurities. In this case, we may assume a Lorentzian broadening of the δ-function in equation (16), i.e. $\delta(E) = (\Gamma/\pi)/(E^2 + \Gamma^2)$, and get

$$D(E) = \frac{1}{2\pi a} \sum_{n=0}^{\infty} \int_0^{a/l^2} \mathrm{d}k_y \frac{\Gamma/\pi}{(E - E_{n,k_y})^2 + \Gamma^2}. \tag{19}$$

Equation (19) contains both the effects of scattering impurities and the modulation broadening of Landau levels. For a non-interacting 2DEG with areal density n_e, the DOS satisfies the relation $n_e = 2\int_0^\infty \mathrm{d}E f(E) D(E)$, where $f(E)$ is the Fermi–Dirac distribution, and 2 is the spin degeneracy factor. With the relation, the Fermi energy E_F of the 2DEG may be determined for a definite magnetic field and it will be an oscillatory quantity with respect to the magnetic field as in the case of potential modulation [9].

The numerical solution of the DOSs are shown in figure 1 for three different values of impurity Landau level broadening: $\Gamma = 0.01$, 0.1 and 0.2 meV at $B_0 = 0.6$ T. Note that the DOSs in figure 1 are plotted in units of their values at zero magnetic field, $D_0 = m^*/\pi\hbar^2$. The solid lines in figure 1 represent the DOSs at lowest order of Fourier transformation, i.e. $R_{p=1} = pK/f(pK) = 1.0$ and $R_{p\geq 2} = 0.0$. The broken curves represent the cases of $R_{p=1,2} = 1.0$ and $R_{p\geq 3} = 0.0$, corresponding to the magnetic modulation $B_1(x) = B_1(\cos[(2\pi/a)x] + \frac{1}{2}\cos[(4\pi/a)x])$.

We can see from figure 1 that for a sufficiently small impurity broadening parameter (small Γ) the DOSs have 1D van Hove singularities at the low- and high-energy edges of each modulation-broadened Landau band, corresponding to a band width larger than the value of Γ. The DOS peaks of different Landau bands are well separated from each other (see figure 1(a)). The wide band corresponds to the lower DOS peaks, and the narrow band to the higher peaks, which is a consequence of the fact that each Landau level contains the same number of states.

For the larger value of Γ, as shown in figure 1(b), the DOSs of different Landau levels start to overlap. The peak heights become small, and the van Hove singularities disappear for some Landau levels. If the impurity broadening is so large that different levels become heavily overlapping, the van Hove singularities disappear completely (see figure 1(c)). So the impurities have very important effects on the properties of DOSs, and therefore on the transport properties of the 2DEG. In the numerical analysis in section 5 we take a relatively small Γ so as to obtain more information on the 2DEG modulated by the magnetic field.

From figure 1 we also noted that the high-frequency components of modulation influence the properties of van Hove singularities of different Landau levels, while they do not change the corresponding band width. Therefore, they will influence the transport properties of the 2DEG, as discussed in section 5.

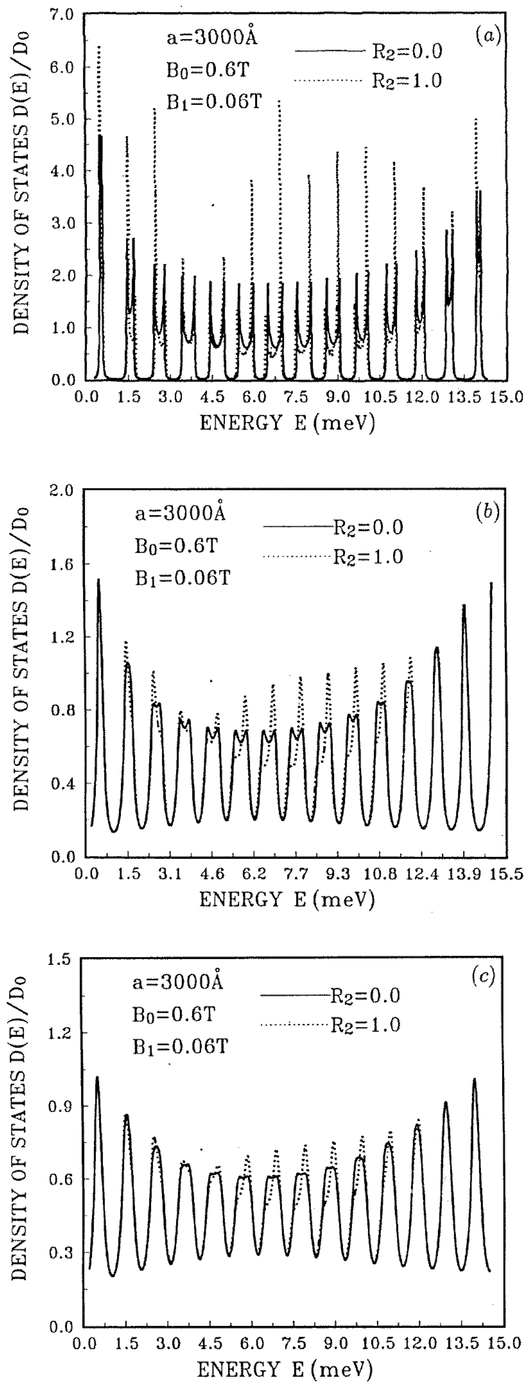

Figure 1. DOSs in units of their values at zero magnetic field, $D_0 = m^*/\pi\hbar^2$, at $B_0 = 0.6$ T and $B_1 = 0.06$ T for (*a*) $\Gamma = 0.01$ meV, (*b*) $\Gamma = 0.1$ meV and $\Gamma = 0.16$ meV.

4. Conductivity tensor

We now consider the conductivity tensor for a 2DEG modulated by a general form of modulation magnetic field. We use the linear response theory [18] due to the weak modulation field to calculate the electrical transport coefficients, as given in [18], derived from the general Liouville equation. For the static modulation and in the one-particle approximation the diagonal components of conductivity tensor consist of the band conduction contribution

$$\sigma^b_{\mu\mu}(0) = \frac{\beta e^2}{A} \sum_{\xi} f_\xi (1 - f_\xi) \tau(E_\xi) |\langle \xi | v_\mu | \xi \rangle|^2 \tag{20}$$

and the scattering contribution to conduction given by

$$\sigma^s_{\mu\mu}(0) = \frac{\beta e^2}{2A} \sum_{\xi\xi'} f_\xi (1 - f_{\xi'}) W_{\xi\xi'} (\alpha^\xi_\mu - \alpha^{\xi'}_\mu)^2 \tag{21}$$

where $\beta = 1/k_B T$, A is the area of the sample, f_ξ is the energy distribution function of electron at state ξ, $\tau(E_\xi)$ is the relaxation time corresponding to the electron state $|\xi\rangle$, $W_{\xi\xi'}$ is the transition rate from $|\xi'\rangle$ to $|\xi\rangle$ as that in equation (18), and $\alpha^\xi_\mu = \langle \xi | r_\mu | \xi \rangle$, where r_μ is the μth component of the electron position operator.

The band conduction (also called diffusion conduction) contribution depends on the group velocities of electrons and is absent in a homogeneous 2DEG. It increases with increasing band width and decreases with increasing impurity scattering, while the scattering conduction (also called collisional conduction) describes the transport through localized states, corresponding to the quantum hopping of the cyclotron motion of electrons and increases with increasing impurity scattering [9, 19].

The static non-diagonal conduction $\sigma^n_{\mu\nu}(0)$ is [18]

$$\sigma_{\mu\nu}(0) = \frac{\mathrm{i}\hbar e^2}{A} \sum_{\xi \neq \xi'} (f_\xi - f_{\xi'}) \frac{\langle \xi | v_\mu | \xi' \rangle \langle \xi' | v_\nu | \xi \rangle}{(E_\xi - E_{\xi'})^2}. \tag{22}$$

In the energy spectrum as discussed above, the spin splittings have not been considered in order to simplify the calculation. Then the spin degeneracy factor (equal to 2) must be included when we use equations (20)–(22) to calculate the conductivity tensor. We assume that electrons in the 2DEG are elastically scattered by the randomly distributed impurities at low temperatures ($T < 10$ K), i.e. for the scattering states $|\xi'\rangle \neq |\xi\rangle$, $f_{\xi'} = f_\xi$. As a further simplification, we also assume that the transport relaxation time $\tau \simeq \mu m^*/e$ (μ is the mobility of an electron at zero magnetic field), independent of the energy of the electrons, which is a reasonable approximation for low magnetic fields ($B_0 < 1.0$ T).

We now calculate the band contribution of the conductivity tensor. Starting from equations (20), (14) and (15), we have

$$\sigma^b_{xx} = 0 \tag{23}$$

$$\sigma^b_{yy} = \frac{2\pi e^2 \tau l^2}{\hbar^2 a^2} \sum_{n=0}^{\infty} \left(-\frac{\partial f}{\partial E} \right) \bigg|_{E=E^0_n} \left[\sum_{p=1}^{\infty} p^2 \epsilon^2_{n,p} \right] \tag{24}$$

to the order of $(\hbar\omega_1)^2$. The $p = 1$ term in equation (24) corresponds to equation (10) of [16].

To evaluate the scattering contribution of conductivity, we firstly calculate and give the relevant matrix elements

$${}^1\langle n k_y | x | n, k_y \rangle^1 = l^2 k_y + \frac{H^1_{n,n-1}}{\hbar\omega_c} \sqrt{2nl^2} + \frac{H^1_{n,n+1}}{(-\hbar\omega_c)} \sqrt{2(n+1)l^2} \tag{25}$$

and

$$ {}^1\langle n'k'_y|\exp(\mathrm{i}\mathbf{q}\cdot\mathbf{r})|nk_y\rangle^1 = {}^0\langle n'k'_y|\exp(\mathrm{i}\mathbf{q}\cdot\mathbf{r})|nk_y\rangle^0 + \sum_{n'\neq m}\frac{H^1_{n',m}}{E^0_{n'}-E^0_m}\,{}^0\langle mk'_y|\exp(\mathrm{i}\mathbf{q}\cdot\mathbf{r})|nk_y\rangle^0 + \sum_{n\neq m}\frac{H^1_{n,m}}{E^0_n-E^0_m}\,{}^0\langle n'k'_y|\exp(\mathrm{i}\mathbf{q}\cdot\mathbf{r})|mk_y\rangle^0 \qquad (26) $$

to the order of $\hbar\omega_1$, where ${}^0\langle n'k'_y|\exp(\mathrm{i}\mathbf{q}\cdot\mathbf{r})|nk_y\rangle^0$ is given by equation (11).

Considering the dominant terms $m = n' \pm 1$ and $m = n \pm 1$ in equation (26), we get the scattering contribution from equations (18) and (21):

$$ \sigma^s_{xx} = \frac{e^2}{h}\frac{n_i V_0^2}{\pi\Gamma a}\sum_{n=0}^{\infty}[(2n+1)A_n + B_n] \qquad (27) $$

to the order of $(\hbar\omega_1)^2$, where

$$ A_n = \int_0^{a/l^2} \mathrm{d}k_y \left[-\frac{\partial f}{\partial E}\right]\bigg|_{E=E_{n,k_y}} \qquad (28) $$

and

$$ B_n = \frac{1}{2}\left(\frac{\omega_1}{\omega_c}\right)^2 \sum_{p=1}^{\infty}(p^2u)\exp(-p^2u)\left[\frac{pK}{f(pK)}\right]^2 \times\{D^2_{n-1,p}(u) + D^2_{n,p}(u) + D_{n-1,p}(u)D_{n,p}(u)\}E_{n,p} \qquad (29) $$

with

$$ D_{n,p}(u) = [1-(p^2u)^{-1}]L^1_n(p^2u) + 2L^2_{n-1}(p^2u) \qquad (30) $$

and

$$ E_{n,p} = \int_0^{a/l^2} \mathrm{d}k_y \sin^2(pKx_0)\left[-\frac{\partial f}{\partial E}\right]\bigg|_{E=E_{n,k_y}}. \qquad (31) $$

Equation (27) is formally the same as equation (13) of [17] and contains a higher order of Fourier components to the second-order perturbation contribution of modulation, absent in [17], which is more important to the oscillating properties of magnetoconductance as discussed in section 5.

In the above calculation, we have taken the approximation that the scattering broadening of Landau level $\Gamma_n = \Gamma$, as that below equation (18). In the absence of magnetic modulation and low-temperature limit, i.e. $B_1(x) = 0$, and $T \to 0$, equation (27) gives $\sigma = (e^2/\hbar\pi^2)(n_F + \frac{1}{2})$, which depends only on the Landau level index n_F and the natural constants and is independent of the magnetic field and the scattering strength. This corresponds to the peaks of ordinary SdH oscillations of 2DEG in a uniform magnetic field [19].

Using the zeroth-order velocity matrix elements [7]

$$ \frac{\sqrt{2}}{l\omega_c}\,{}^0\left\langle n, k_y \left| \begin{matrix} -\mathrm{i}v_x \\ v_y \end{matrix} \right| n', k_y\right\rangle^0 = \sqrt{n+1}\,\delta_{n',n+1} \mp \sqrt{n}\,\delta_{n',n-1} \qquad (32) $$

the Hall conductivity $\sigma_{yx}(= -\sigma_{xy})$ can be evaluated with equation (22). The result is

$$ \sigma_{yx} = \frac{2e^2}{h}\frac{l^2}{a}\sum_{n=0}^{\infty}(n+1)\int_0^{a/l^2}\mathrm{d}k_y \frac{f_{n,k_y} - f_{n+1,k_y}}{\left(1+\sum_{p=1}^{\infty}\lambda_{n,p}\cos(pKx_0)\right)^2} \qquad (33) $$

where $\lambda_{n,p} = [\epsilon_{n+1,p} - \epsilon_{n,p}]/\hbar\omega_c$ with $\epsilon_{n,p}$ given by equation (13); the $p = 1$ term is the same as equation (16) of [17]. The first-order term, linear in $\hbar\omega_1$, is absent in the numerator of the integrand, different from that of [16]. In the absence of modulation ($\lambda_{n,p} = 0$) and low-temperature limit ($T \to 0$), we have the Hall conductivity $\sigma_{yx} = Ne^2/h$, where N is positive integers. This corresponds to the plateau value of the quantized Hall effect of 2DEG in a uniform magnetic field [19], where the transport across the magnetic superlattice is without dissipation, as discussed in the next section.

5. Numerical results and discussion

The numerical analysis can proceed for the components of the conductivity tensor from equations (23), (24), (27) and (33). We take as an example the high-mobility $Al_xGa_{1-x}As$–GaAs heterojunction ($m^* = 0.067m$), with the areal carrier density $n_e = 3.16\times10^{15}$ m^{-2}, the electron mobility $\mu_e = 1.3\times10^{10}$ m^2 s^{-1}, the impurity concentration $n_i = 1\times10^{12}$ m^{-2} and a small impurity broadening $\Gamma = 0.0129\sqrt{B_0}$ meV. For the 1D magnetic modulation with $a = 3000$ Å and $B_1 = 0.06$ T in the lowest-order approximation of Fourier transformation, i.e. $B_1(x) = B_1\cos[(2\pi/a)x]$, corresponding to $R_{p\geq2} = pK/f(pK) = 0$, the numerical results are shown in figure 2(a) for two temperatures $T = 1$ K and $T = 6$ K. Note that in the component σ_{xx} contains only the scattering contribution, while σ_{yy} contains both the scattering contribution ($\sigma^s_{yy} = \sigma^s_{xx}$) and the band conduction (σ^b_{yy}). The modulation correction to the conductivity, $\Delta\sigma_{\mu\nu} = \sigma_{\mu\nu}(B_1) - \sigma_{\mu\nu}(B_1 = 0)$, is also evaluated as shown in figure 2(b) for $R_p = pK/f(pK) = 0\,(p \geq 2)$. In the case of $R_2 = 1.0$ and $R_{p\geq3} = 0$, i.e. $B_1(x) = B_1\{\cos[(2\pi/a)x] + \frac{1}{2}\cos[(4\pi/a)x]\}$, $\Delta\sigma_{\mu\nu}$ is shown in figure 2(c) for the temperature $T = 6$ K. Note that, in figures 2(b) and 2(c), $\Delta\sigma_{\mu\nu}$ are plotted offset and the zero points are indicated by the corresponding horizontal arrows. σ_{yx} has been rescaled so as to plot the different components of conductivity tensor in the same figure.

Figure 2 shows that the 1D magnetic modulation gives the following features of conductivity tensors.

(1) At low temperatures, there are two kinds of oscillation: the new oscillations (Weiss oscillations) and the SdH-type oscillations. The Weiss oscillations stem from the oscillatory band width of the modulation-broadened Landau level at the Fermi energy E_F, while the SdH oscillations stem from the oscillatory DOS at E_F. The period T_{WOS} of Weiss oscillations is much larger than the period T_{SdH} of the corresponding SdH oscillations, i.e. $T_{WOS} \gg T_{\mathrm{SdH}}$. Therefore, at $B_0 \leq 0.2$ T, only the Weiss oscillations appear since the SdH oscillations are too weak to be resolved while, at $B_0 > 0.2$ T, the SdH oscillations appear and overlap the slowly oscillatory envelopes (Weiss oscillations) for $T = 1$ K.

(2) $\Delta\sigma_{yy}$ is positive, while $\Delta\sigma_{xx}$ and $\Delta\sigma_{yx}$ oscillate around the zero point.

(3) $|\Delta\sigma_{yy}| \gg |\Delta\sigma_{xx}|$. This is because the band contribution in σ_{yy} is much greater than that of the scattering contribution, which is the unique part that σ_{xx} contains.

(4) The Weiss oscillations in σ_{yy} (and $\Delta\sigma_{yy}$) and σ_{xx} (and $\Delta\sigma_{xx}$) are out of phase since $\epsilon_{n,1}$ and the corresponding DOSs are out of phase, while $\Delta\sigma_{yx}$ and $\Delta\sigma_{xx}$ oscillate in phase.

(5) At $B_0 > 0.2$ T, the SdH oscillations are in phase in σ_{xx} and σ_{yy} and out of phase in $\Delta\sigma_{xx}$ and $\Delta\sigma_{yy}$, while the oscillation in $\Delta\sigma_{yx}$ is shifted by 90° with respect to that in $\Delta\sigma_{xx}$.

(6) At larger fields, $B_0 > 0.6$ T, we have $\sigma_{yy} \gg \sigma_{xx}$. This reflects the fact that the scattering contribution decreases with the increasing magnetic field.

(7) At higher temperatures $T = 6$ K, the SdH oscillations are damped out, only Weiss oscillations exist. Weiss oscillations have a weaker dependence on the temperature in

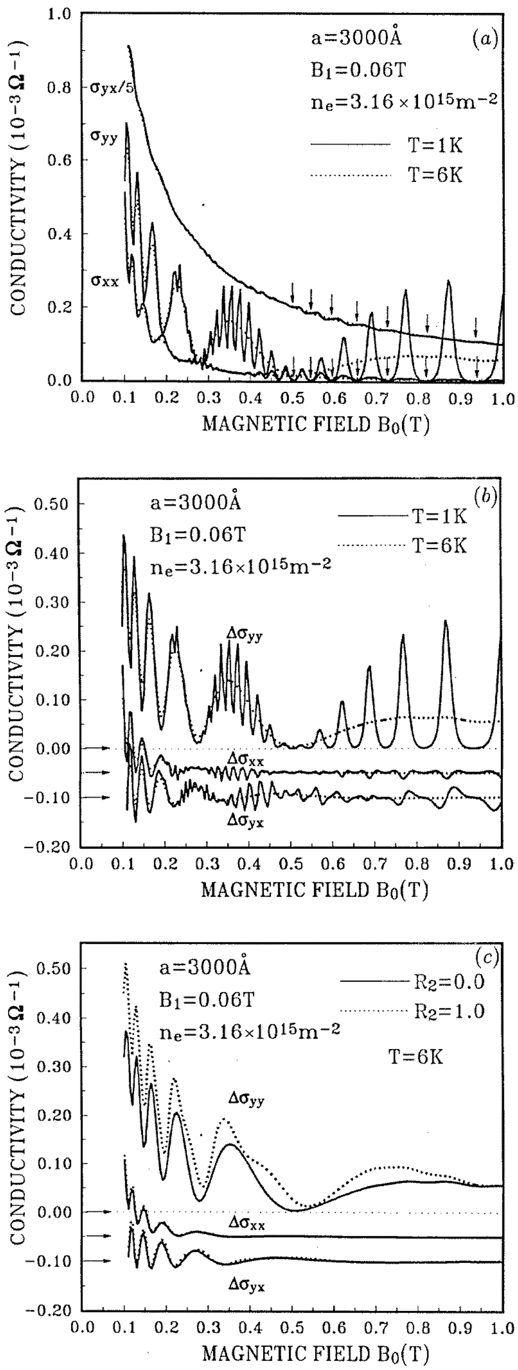

Figure 2. (*a*) The calculated conductivity tensor $\sigma_{\mu\nu}$ versus the uniform magnetic field B_0. (*b*) The modulation correction to the conductivity tensor $\Delta\sigma_{\mu\nu}$. (*c*) $\Delta\sigma_{\mu\nu}$ for the modulation $B_1(x) = B_1\{\cos[(2\pi/a)x] + \frac{1}{2}\cos[(4\pi/a)x]\}$ at $T = 6$ K. The dotted lines in (*a*) and (*b*) represent the zero points of $\Delta\sigma_{yy}$. The short vertical arrows represent the Hall plateaux and the corresponding minima of σ_{yy}.

contrast with the sensitive dependence of SdH oscillations.

(8) For the modulation with high-frequency components, as shown in figure 2(c), the Weiss oscillations manifest themselves non-monoperiodically at $B_0 > 0.35$ T, while they remain monoperiodic at $B_0 < 0.35$ T. This indicates that the contribution of high-frequency components in modulation ($p \geq 2$ as in equation (3)) is much weaker than that of the base frequency in modulation ($p = 1$). The theoretical interpretation is that $\epsilon_{n,p}$ contains the exponential factor $\exp(-p^2 u)$. For high-frequency components, $p \geq 2$, the ratio of the corresponding contribution to that of the low-frequency component is $\exp[-(p^2 - 1)u]$, which is much smaller than unity at low magnetic fields where $u(= \frac{1}{2}K^2 l^2 \propto 1/B)$ is large. Therefore, at low fields, $\sigma_{\mu\nu}$ oscillates with a unique period (in $1/B$) while, at high fields, the ratio increases with increasing field and is comparable with the unity. Then $\sigma_{\mu\nu}$ oscillates with multiple-period components.

The resistivity tensor $\rho_{\mu\nu}$ can be evaluated in terms of the conductivity tensor $\sigma_{\mu\nu}$. From $\rho\sigma = 1$, we have $\rho_{xx} = \sigma_{yy}/S$, $\rho_{yy} = \sigma_{xx}/S$, and the Hall resistivity $\rho_{xy} = \sigma_{yx}/S$, where $S = \sigma_{xx}\sigma_{yy} + \sigma_{yx}^2$. In the numerical evaluation, we have taken the following approximation. From figure 2, we know that firstly $\sigma_{yx} \gg \sigma_{xx}$ and σ_{yy} and secondly $|\Delta\sigma_{yx}| \ll \sigma_{yx}$. Then we may use $S \approx \sigma_{yx}^2 \approx n_e e/B_0$ to calculate ρ_{xx} and ρ_{yy} while for the Hall resistivity we have $\rho_{xy} = 1/\sigma_{yx}$. The numerical results of the resistivity tensor and the contribution due to modulation ($\Delta\rho_{\mu\nu} = \rho_{\mu\nu}(B_1) - \rho_{\mu\nu}(B_1 = 0)$) are shown in figures 3(a) and 3(b) for $T = 1$ K and $T = 6$ K at the lowest order of approximation of Fourier transformation ($R_p = 0$, for $p \geq 2$). Note that ρ_{xx} and $\Delta\rho_{xx}$ have been rescaled so as to plot the different components of the resistivity tensor in the same figure. From figures 3(a) and 3(b), we obtain the following.

(1) At $B_0 < 0.2$ T, there are only Weiss oscillations in $\rho_{\mu\nu}$ while, at $B_0 > 0.2$ T, SdH oscillations appear and overlap Weiss oscillations for $T = 1$ K, where the conditions $\hbar\omega_c \gg k_B T$ and $\hbar\omega_c \gg \Gamma$ are satisfied.

(2) Weiss oscillations in ρ_{xx} (and $\Delta\rho_{xx}$) and ρ_{yy} (and $\Delta\rho_{yy}$) are 180° out of phase, since the resistivity ρ_{yy} is determined by the values of the DOS at the Fermi energy E_F, while ρ_{xx} is mainly determined by the width of the Landau level at E_F. In contrast, $\Delta\rho_{yx}$ oscillates in phase with $\Delta\rho_{xx}$.

(3) The SdH oscillations are in phase in ρ_{xx} and ρ_{yy} and out of phase in $\Delta\rho_{xx}$ and $\Delta\rho_{yy}$. In $\Delta\rho_{yx}$ the SdH oscillations are shifted by 90° with respect to that in $\Delta\rho_{yy}$.

(4) $|\Delta\rho_{xx}| \gg |\Delta\rho_{yy}|, |\Delta\rho_{xy}|$.

(5) The modulation gives an overall non-negative contribution (but with a series minima of nearly zero values, as discussed below) to ρ_{xx}, i.e. $\Delta\rho_{xx} \geq 0$ for $B_0 = 0.1$–1.0 T.

We note that ρ_{yy} has obvious Weiss oscillations, in contrast with the result of [16], where ρ_{yy} has no Weiss oscillations at low fields. This is because we have considered the second order of perturbation of modulation in σ_{xx} (in equation (27)), which is more important to the oscillating properties of ρ_{yy}.

The contribution to the resistivity ρ_{xx} of high-frequency Fourier components of 1D modulation is shown in figure 3(c) for temperatures $T = 1$, 2, 4 and 6 K. Note that, at $B_0 > 0.2$ T, the Weiss oscillations have multiple-frequency components. Figure 3(c) also shows the much weaker dependence of Weiss oscillations on temperature, compared with SdH oscillations. At high temperatures ($T \geq 6$ K), the SdH oscillations are smeared out.

Figures 2(a) and 3(a) also show the surprising result that the Hall conductivity σ_{yx} (resistivity ρ_{xy}) displays quantized plateaux whenever $\sigma_{yy}(\rho_{xx})$ has pronounced minima as in the case of potential modulation [14], which are close to zero (indicated by the short vertical arrows). We know that a modulated magnetic field created by strips of ferromagnetic

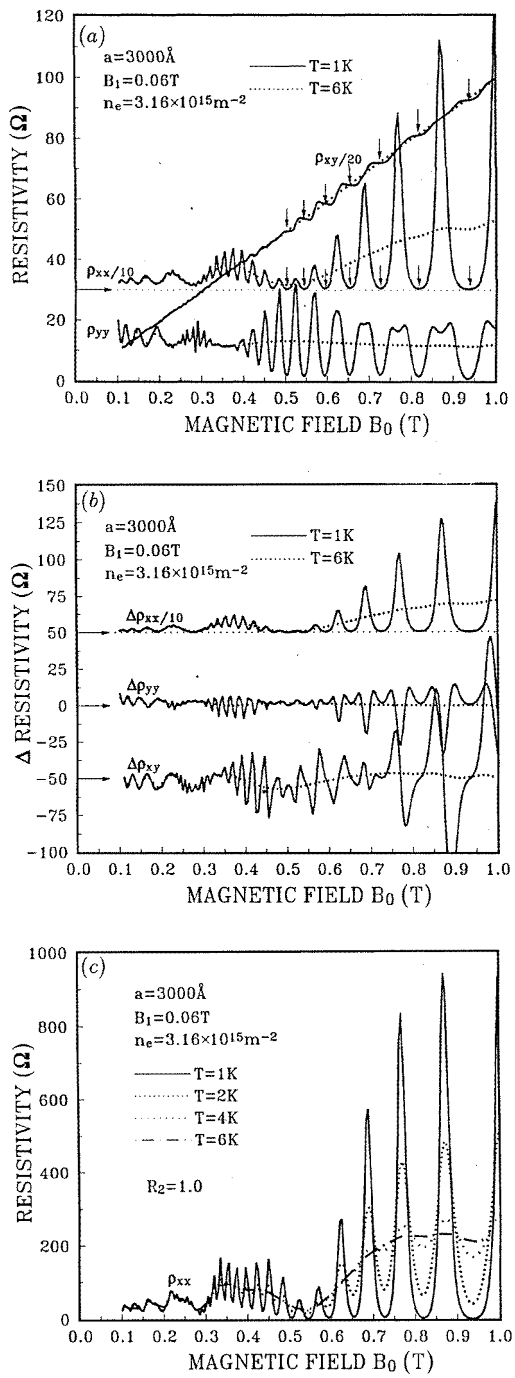

Figure 3. (*a*) The calculated resistivity tensor $\rho_{\mu\nu}$ versus the uniform magnetic field B_0. (*b*) The modulation correction to the resistivity tensor $\Delta\rho_{\mu\nu}$. (*c*) $\Delta\rho_{xx}$ for the modulation $B_1(x) = B_1\{\cos[(2\pi/a)x] + \frac{1}{2}\cos[(4\pi/a)x]\}$ at four different temperatures. The lines in (*a*) and (*b*) represent the zero points of ρ_{xx} and $\Delta\rho_{xx}$, respectively. The short vertical arrows represent the Hall plateaux and the corresponding minima of ρ_{xx}.

or superconducting material can be presented as an effective series of potential barriers [20]. It is also well known that the resistance measured across a wide barrier is finite. Why is the electric transport across the superlattice structure nearly dissipationless? This can be explained in terms of the Büttiker edge channel picture, as discussed in [14] for potential modulation. In the edge channel picture the finite resistance is due to the back scattering of electrons from one side of the sample to the other [14]. In the quantum Hall regime, transport is described by the 1D edge states, located at the boundaries of the 2DEG where bent Landau levels cross the Fermi level E_F, while in the edge states the Landau level filling factor $\nu = 2\pi l^2 n_e$. Electrons can either move along the barrier or be transmitted through the barrier. Corresponding to the integer filling factor ν (Hall plateaus), σ_{xx} is finite (as shown in figure 2(a)) and the barriers have a finite transmission. In this case, the edge channel will 'leak' to the other side of the barrier and return to the same edge of the sample [14]. Owing to the huge ratio of barrier length to barrier width (the typical value is about 10^3), an electron is transmitted through the barrier long before it reaches the other side of the 2DEG and there is no back scattering along the barrier. This is the reason why the transport across the magnetic barrier occurs without dissipation.

The Weiss oscillations and the SdH oscillations are more pronounced in the $B_0 - \mathrm{d}R_H/\mathrm{d}B$ graph, as shown in figure 4(a) for the lowest order of Fourier transformation. For $T = 1$ K, the SdH oscillations appear at $B_0 > 0.11$ T. In figure 4(b), we give the same plots for the modulation field $B_1(x) = B_1\{\cos[(2\pi/a)x] + \frac{1}{2}\cos[(4\pi/a)x]\}$. In this case, Weiss oscillations have obvious multiple-frequency components at $B_0 > 0.17$ T. In experiments, $\mathrm{d}R_H/\mathrm{d}B$ can be determined from the measurement of the corresponding $\Delta\rho_{xy}$.

Recently, experimental measurements on a 2DEG modulated by a 1D periodic magnetic field have been reported by two groups for different modulation strengths with a relatively larger modulation period ($a \simeq 1$ μm) [21, 22]. They observed the new oscillations in the magnetoresistance, in addition to SdH oscillations, which correspond to the commensurability between the semiclassical cyclotron diameter and the period of magnetic modulation. Owing to the unavoidable presence of the weak electrostatic potential modulation due to the differential contraction of the magnetic material (which is responsible for the magnetic modulation) and the substrate, the SdH oscillations are not obvious in the experimental results and can only be observed when the magnetic field modulation dominates over the weak electric potential [21, 22]. The experimental results are in qualitative agreement with our theory, while the quantitative comparison is very difficult owing to the presence of potential modulation in the experiments and the different modulation periods in experiments and our theory. The phenomenon of dissipationless transport across the magnetic superlattice is not present in the experiments [21, 22] as given in our theory above.

6. Conclusion

We have given a detailed investigation of the energy spectrum and the electric properties of the 2DEG for a general form of the weak 1D periodic magnetic field in the one-particle approximation to the order of $(\hbar\omega_1)^2$, including higher-order components of Fourier transformation, by generalizing the theory of Peeters and Vasilopoulos [15, 17] and that of Xue and Xiao [16] on the magnetic modulation in the lowest order of approximation of Fourier transformation. This modulation broadens the Landau levels into bands and leads to band conduction along the direction perpendicular to the modulation, which is absent in the uniform magnetic field, where only the quantum hopping of cyclotron motion exists. The strength of the band conduction is proportional to the sum of the square of all the different

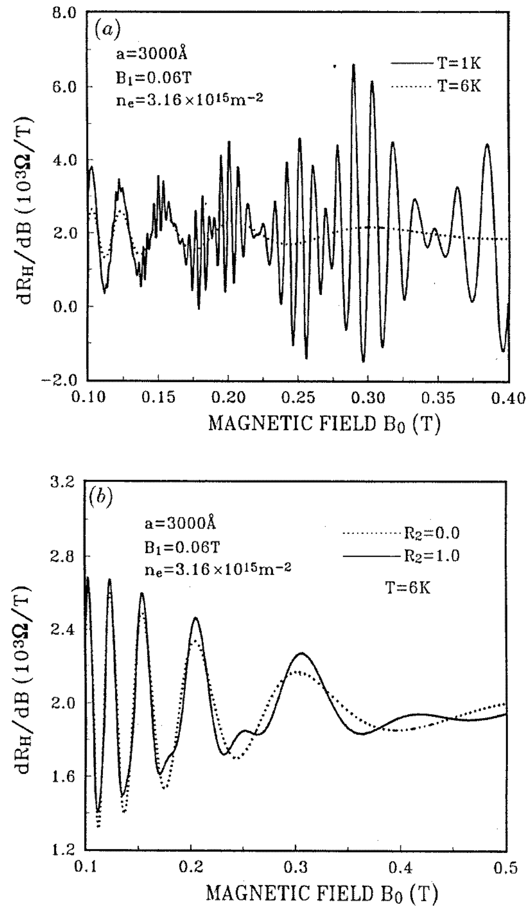

Figure 4. (*a*) The derivative of Hall resistance versus the uniform magnetic field B_0 for $T = 1$ K (——) and for $T = 6$ K (······). (*b*) The same as (*a*) at $T = 6$ K for $B_1(x) = B_1\{\cos[(2\pi/a)x] + (R_2/2)\cos[(4\pi/a)x]\}$ at $T = 6$ K in the case of $R_2 = 0.0$ (······) and $R_2 = 1.0$ (——).

components of Fourier transformation in modulation. The electric transport coefficients $\sigma_{\mu\nu}$ and $\rho_{\mu\nu}$ have Weiss oscillations due to the oscillatory band width at the Fermi energy E_F (resulting from the modulation) and SdH oscillations due to the oscillatory DOSs at E_F, similar to the case of a potential-modulated 2DEG. ρ_{xx} has a much larger oscillating amplitude than ρ_{yy}, due to the existence of band conduction in ρ_{xx}, which is much larger than the scattering contribution in the high-mobility sample. The properties of Weiss oscillations and SdH oscillations are more pronounced in $\mathrm{d}R_H/\mathrm{d}B$ as shown in figure 4. Owing to the existence of the exponential factor $\exp(-p^2u)$ in the energy spectrum, the contribution of high-frequency components of modulation is visible only at high fields ($B_0 > 0.5$ T). At low fields, Weiss oscillations have only a single-frequency component, corresponding to the modulation field $B_1(x) = B_1\cos[(2\pi/a)x]$. Therefore, the lowest-order approximation of Fourier transformation may be a good approximation at low fields.

We note that for equal modulation strengths in potential V_0 [9, 17] and in a periodic magnetic field $\hbar\omega_1$, i.e. $V_0 = \hbar\omega_1$, the oscillation amplitude of $\rho_{\mu\nu}$ in the magnetic case is much larger than that in the electric case. We also get the surprising result that, while the Hall resistance displays quantized plateaux, electrons are transported across the magnetic superlattice dissipationless.

The study of the electrical properties of the 2DEG modulated by a periodic magnetic field is of great importance to study the nature of flux lines in type-II superconductors. As suggested by Bending *et al* [23], a 2DEG may be used as a detector of flux-lattice properties. By extending the analysis in this paper to a 2D modulation of magnetic field with square geometry or hexagonal geometry, we may give a more exact interpretation of the experimental results of Kruithof *et al* [24] on the basis of the work in [25] for the case $B_0 \lesssim B_{c2}$, where B_{c2} is the upper critical field of the type-II superconductor.

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